Constant Acceleration Equations

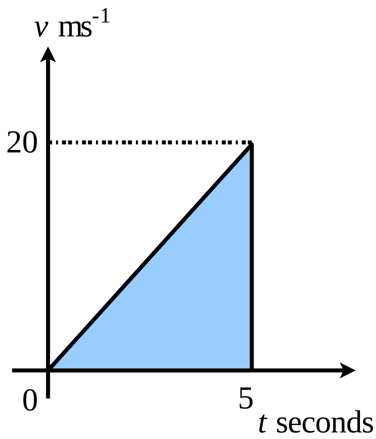
Acceleration

Acceleration is the **rate of change of velocity of a moving object**.
It is usually measured in 'metres per second squared', which is abbreviated to $\mathbf{ms^{-2}}$.
An acceleration of 2 ms^{-2} means that the velocity of the object is increasing by 2 metres per second every second. If the velocity of the object is decreasing by 2 metres per second every second, then the acceleration is – 2 ms^{-2}. This is called deceleration or retardation.

Like velocity, acceleration is a **vector**, having both direction and magnitude.
(Note a negative sign may also indicate the object is speeding up in the negative direction.)

Velocity-time graphs

Acceleration
The gradient of the velocity-time graph of a moving object gives its acceleration.
If the graph is a straight line the acceleration is constant.



Example
This graph shows the velocity of a car that accelerates at a constant rate from 0 to 20 ms^{-1} in 5 seconds.

The car's acceleration = $\dfrac{20}{5}$ = 4 ms^{-2}

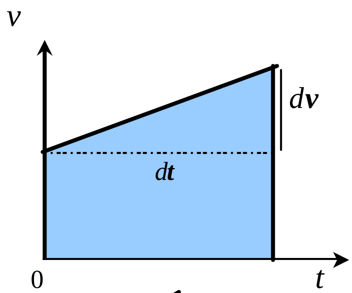
You may know that the area under a velocity-time graph gives the displacement or distance moved.

In this case the distance moved = $\dfrac{5 \times 20}{2}$ = 50 m

The facts you may need in the following work are summarised below. Use them on the next page to derive an important set of equations that apply when objects move with constant acceleration. These equations can be used to model many real-life situations.

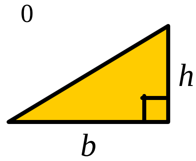
Summary

Gradient of a velocity-time graph = $\dfrac{dv}{dt}$ **= acceleration**



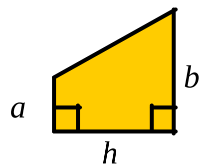
Area under a velocity-time graph = displacement

Area of a triangle $= \dfrac{\text{base} \times \text{height}}{2} = \dfrac{bh}{2}$



Area of a trapezium $= \dfrac{\text{sum of parallel sides} \times \text{distance between}}{2}$

$= \dfrac{(a+b)h}{2}$



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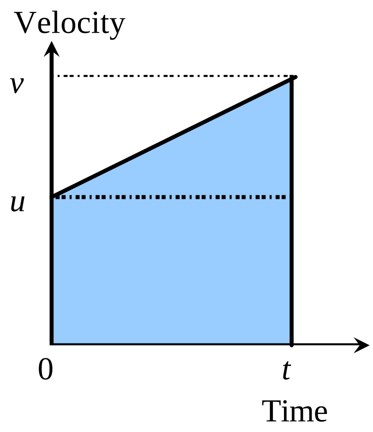
Constant Acceleration Equations

This velocity-time graph shows the velocity of an object increasing from an initial value, u, to a final value, v, in time t. Work through the following steps to derive the equations.



Write down an expression for the acceleration, a, in terms of u, v and t.

………… $a =$ ………………………………………………

Rearrange this equation to make the subject v.

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………… $v =$ ……………………………………………(1)

Use the area under the graph to write an expression for the displacement, s, in terms of u, v and t.

………… $s =$ ……………………………………………(2)

Substitute the expression for v from equation (1) into equation (2) and rearrange to give equation (3) below.

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………… $s = ut + \frac{1}{2}at^2$ ……………………………(3)

Rearrange equation (1) to give an expression for t, then substitute this into equation (2) and rearrange to give equation (4) below.

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………… $v^2 = u^2 + 2as$ ……………………………(4)

The equations for motion in a straight line with constant acceleration are:

$v = u + at$ ……….(1)

$s = \dfrac{(u+v)t}{2}$ ……...(2)

$s = ut + \frac{1}{2}at^2$ …....(3)

$v^2 = u^2 + 2as$ …..(4)

where
u is the initial velocity
v is the final velocity
a is the acceleration
t is the time taken
s is the displacement

Teacher Notes

Unit Advanced Level, Dynamics

Notes

This activity allows learners to derive the constant acceleration equations.

They will need to know that:

- the gradient of a velocity-time graph gives the acceleration
- the area under a velocity-time graph gives the displacement
- the area of a trapezium = $\dfrac{\text{sum of parallel sides} \times \text{distance between}}{2} = \dfrac{(a+b)h}{2}$

These facts are given on page 1 and also on slides 1 to 3 in the Powerpoint presentation. Slides 4 to 7 give the derivation of the constant acceleration equations - these can be used as a demonstration or to check learners' results.